DR. BABASAHEB AMBEDKAR MARATHWADA UNIVERSITY

NACC Re-Accredited ‘A’ Grade

DEPARTMENT OF MATHEMATICS

SYLLABUS

FOR

Entrance Test

For the Admission to the Course

M.Phil. in Mathematics

For the Academic Year

2019-2020

Advanced Abstract Algebra- I

Unit- I Binary relation, binary operation, function, group, subgroup and their properties. Order of a group. Generator, cyclic group, Lagranges theorem, Fermats and Eulers theorem and their consequences.

Unit- II Normal subgroup, quotient group and their properties and examples. Homomorphism, kernel, image of a homomorphism. Isomorphism and related theorems, Fundamental theorem of group homomorphism, automorphism, conjugacy and G-sets.

Unit- III Permutation groups and related concepts and results. Center, normalizer,commutator of a group, derved group, Cayles theorem.

Unit – IV Normal series, solvable and nilpotent group and their properties, direct products, simplicity of alternating group.

Unit- V Fundamental theorem of finitely generated abelian group, invariants of a finite abelian group,

Real Analysis- I

Unit – I Definition and existence of Riemann-Stieltjes integral, Properties of the integral, Integration and Differentiation, The fundamental theorem of calculus, Examples.

Unit – II Integration of vector valued functions. Rectifiable curve. Examples. Sequences and series of functions. Point wise and uniform convergence. Cauehy criterion for uniform convergence. Weierstrass M-test, uniform convergence and continuity, uniform convergence and Riemann-Stielljes integration. Examples.

Unit – III Uniform convergence and Differential, The Stone – Weierstrass theorem, Examples. Power series, Abel’s and Taylor’s theorems, Uniqueness theorem for power series. Examples.

Unit – IV Functions of several variables, Linear transformations, Derivatives in an open subset of R^n, Chain rule, Examples

Unit – V Partial derivations. Interchange of the order of differentiation, The inverse function theorem, The implicit function theorem Jacobins, Derivatives of higher order, Differentiation of integrals. Examples,

Topology - I

Unit I **:** Recall definitions of functions, images and inverse images of sets under given mappings, metric spaces, open disks in metric spaces, open and closed sets and their properties, continuity and its formulations in terms of open and closed sets.

Unit II **:** Introduction to topology and topological spaces, open sets, closed sets, closure, interior, neighborhoods, neighborhood systems, neighborhood bases at a point, weaker and stronger topologies, the Hausdorff Criterion, cluster point, derived set.

Unit III **:** Base for a topology, subbase for a topology, criterion for base, subspace of a topological spaces, nature of open sets, closed sets, Neighborhoods in subspaces, continuous functions on topological spaces and criterions of continuity, homeomorphism.

Unit IV**:** Product spaces, box topology on finite Cartesian product, Tychonoff topology or product topology on a general product, evaluation maps, quotient topology, quotient spaces, sequences in topological spaces and their inadequacy.

Unit V**:** Net, convergence of net, cluster point of a net, subnet, continuity of functions in terms of net, ultranet, Filters and their convergence, continuity of functions in terms of filters, ultrafilter, relation between nets and filters.

Complex Analysis – I

Unit- I: The Complex number system: The field of complex numbers, The complex plane, Rectangular and polar representation of complex numbers; Intrinsic function on the complex field; The Complex plane.

Unit – II Metric spaces and Topology of C: Definition and examples of metric spaces; connectedness; sequence and completeness; compactness; continuity; Uniform convergence.

Unit- III: Elementary properties and examples of Analytic functions:

Power series; The exponential function; Trigonometric and hyperbolic functions; Argument of nonzero complex number; Roots of unity; Branch of logarithm function. Analytic functions; Cauchy Riemann Equations; Harmonic function;

Unit-IV: Analytic functions as a mapping; Mobius transformations; linear transformations; The point at infinity; Bilinear transformations, Complex Integration: power series representation of analytic functions; zeros of an analytic function.

Unit–V: The index of a closed curve; Cauchy's theorem and integral formula; Gaursat's Theorem; Singularities: Classification of singularities; Residues; The argument principle.

Differential Equations – I

Unit – I Existence, uniqueness and Continuation of solutions: Introduction, Method of successive approximations for the initial value problem y'= f (x,y), $y(x_0)=y_0$, The Lipschitz condition. Peano's existence theorem, maximal and minimal solutions.

Unit – II Continuation of solutions, Existence theorems for system of differential equations: Picard-Lindelof theorem, Peano's existence theorem, Dini's derivatives, differential inequalities.

Unit – III Integral Inequalities: Gronwall- Reid-Bellman inequality and its generalization, Applications: Zieburs theorem, Peron's criterion, Kamke's uniqueness theorem.

Unit – IVLinear systems: Introduction, superposition principle, preliminaries and Basic results, Properties of linear homogeneous system, Theorems on existence of a fundamental system of solutions of first order linear homogeneous system, Abel-Liouville formula.

Unit – V Adjoint system, Periodic linear system, Floquet's theorem and its consequences, Applications, Inhomogeneous linear systems, applications.

Advanced Abstract Algebra –II

Unit- I Preliminaries of rings, subring, ideal, prime, maximal ideas, nil, nilpotent ideals and their properties. Quotient ring, Homomorphism, isomorphism and related results. UFD, PID, Euclidean domain, polynomial rings and their properties.

Unit – II Vector spaces, subspaces, generating set, linear dependence and independence, basis and dimension, quotient space, homomorphism, dual space, inner product space and modules. Liner transformation and their properties,

Unit – III Extension fields, irreducible polynomials, algebraic extension and their properties, splitting field, normal extension, multiple roots, finite fields.

Unit – V Automorphism groups, fixed field, fundamental theorm of Galois theory, polynomials solvable by radicals.

Real Analysis –II

Unit – I Measure on the real line. Lebegue outer measure, measurable sets. Regularity. Measurable functions. Borel and Lebegue measurability. Examples.

Unit – II Integration of functions of a Real variable. Integration of a simple function. Integration of non-negative functions. The general integral. Integration of series. Examples.

Unit – III Riemann and Lebeque Integrals, Differentiation. The four derivates, Functions of bounded variations. Lebegue's differentiation theorem, Examples.

Unit – IV Abstract Measure spaces. Measures and outer measures Extension of a measure. Uniqueness of the extension. Completion of a measure spaces. Integration with respective to a measure. Examples.

Unit – V The L^p spaces. Convex functions. Jensen's inequality. The inequalities of Holder and Minkowski Completeness of L^p (μ) Convergence in measure. Almost uniform convergence. Examples.

Topology -II

Unit I : Separation axioms T_0, T_1, T_2, regularity and complete regularity, T_3 and Tychonoff spaces

Unit II : Normality and its criterions, T_4-spaces, Urysohn's lemma, Countability axioms, first countability, second countability, Lindelof spaces, separable spaces

Unit III : Compactness and its characterizations, compactness together with Hausdorffness and regularity, Locally compact spaces, compactification of topological spaces.

Unit IV: Refinement, star-refinement, barycentric refinement, locally finite collection, point-finite collection, paracompact spaces, metrization of topological spaces.

Unit V: Disconnected spaces, connected spaces, mutually separated sets and criterion of connectedness in terms of them, components, simple chain, Pathwise connected spaces, arcwise connected spaces, locally connected spaces.

Complex Analysis –II

Unit – I Compactness and convergence in the space of Analytic functions:

Spaces of analytic functions; The weierstrass factorization theorem; factorization of the sine function; The gamma function; The Riemann zeta function.

Unit – II Harmonic functions: Basic properties of Harmonic functions and comparison with analytic function; Harmonic functions on a dick; Poisson integral formula; positive harmonic functions.

Unit – III Entire functions; Jensen's formula; The Poisson-Jenson formula; The genus and order of an entire function. Hadamard factorization Theorem.

Unit – IV Univalent functions; the class S; the class T; Bieberbach conjucture; sub class

of s.

Unit – V Analytic continuation: Basic concepts; special functions.

Differential Equations - II

Unit- IPreliminaries, Basic Facts: Superposition principles, Lagrange Identity, Green'sformula, variation of constants, Liouville substitution, Riccati equations Prufer

Transformation. Higher order linear equations.

Unit – II Maximum Principles and their extensions, Generalized maximum principles, initial value problems, boundary value problems.

Unit –III Theorems of strum; sturm's first comparison theorem, sturm's separation theorem, strums second comparison theorem.

Unit – IV Sturm-Liouville boundary Value Problems: definition, eigenvalues, eigenfunctions, orthogonality.

Unit – V Number of zeros, Non oscillatory equations and principal solutions, Nonoscillation theorems.

Functional Analysis

Unit I : Definition of normed linear spaces, Banach spaces, continuity of norm, joint continuity of vector addition and scalar multiplication in normed linear spaces, quotient spaces.

Unit II : Continuous linear transformations and different criterions of continuity of linear transformations on normed linear spaces, space of bounded linear transformations, isometric isomorphism, equivalent norms, Conjugate spaces, Hahn-Banach theorem and its consequences, natural imbedding of normed linear space into its second conjugate.

Unit III : The Open Mapping theorem, projections on Banach spaces, the Closed graph theorem, the Uniform Boundedness theorem, conjugate of an operator, Inner product spaces, Schwarz's inequality, joint continuity of an inner product, parallelogram law in inner product spaces.

Unit IV: Hilbert spaces, Orthogonal complements, Orthonormal sets, Bessel's inequality, conjugate space of a Hilbert space, adjoint of an operator, self-adjoint operators, normal and unitary operators.

Partial Differential Equations

Unit-I First order partial differential equation, linear equations of the first order, integral surface passing through a curve, surfaces orthogonal to a given system of surfaces.

Unit-II Non-linear partial differential equations of the first order, Cauchy's method of characteristics, compatible system of first order equations (condition of compability), Charpit's method.

Unit-III Special types of first order equations, solutions satisfying given conditions,

a) Integral surface through a curve. (b) Derivation of one complete integral from another.
(c) Integral surfaces circumscribing a given surfaces. Jocobi's method for solving $F(x, y, z, p, q) = 0$.

Unit-IV The origin of second order equations, linear partial differential equations with constant coefficients, intermediate integrals or first integrals, Monge's method of integrating $Rr + Ss + Tt = V$, classification of second order partial differential equation (Canonical form).

Linear Integral Equations

Unit I : Definition of Integral Equations and Linear Integral Equations, Types of Linear Integral Equations, Special kinds of Kernels: Separable or degenerate kernel, symmetric kernel, convolution-type kernels, Eigenvalues and eigenfunctions of kernels, Solution of linear integral equations, Verification of solution of linear integral equations, Conversion of Boundary Value Problem to integral equations and vice-versa, conversion of Initial Value Problems to integral equations and vice-versa.

Unit II : Methods of obtaining solution for Fredholm integral equations, Fredholm integral equations with separable kernels, Approximating kernels by separable kernels, Method of successive approximation, Iterated kernel method for Fredholm integral equations, Resolvant kernels and their properties, Methods of solutions for Volterra integral equations, Volterra type kernel, Method of differentiation, Method of successive approximations, Method of iterative kernels, Resolvant kernels and its use to solve Volterra integral equations.

Unit III : Symmetric kernel, trace of a kernel, Fredholm operator, Fundamental properties of symmetric kernels, Eigenvalues and eigenfunctions of symmetric kernel and their properties, normalized eigenfunctions, Iterated kernel of symmetric kernels and their properties, Truncated kernel of symmetric kernel and necessary and sufficient condition for symmetric kernel to be separable, The Hilbert-Schmidt theorem, Method of Solution for Integral equations with symmetric kernels.

Unit IV: Integral Transform Methods, Recall of Laplace and Fourier Transforms, Application of Laplace transform to Volterra integral equations with convolution-type kernel and examples, Application of Fourier transform to some singular integral equations and examples.

Mechanics

Unit-I Mechanics of system of particles, generalized coordinates, Holonomic & nonholonimic system, Scleronomic & Rheonomic system, D' Alemberts's principle and Lagrange's equation of motion, different forms of Lagrange's equation, Generalized potential, conservative fields and its energy equation, Application of Lagrange's formulation.

Unit – II Functionals, Linear functionals, Fundamental lemma of Calculus of Variations simple variational problems, The variation of functional, the extermum of functional, necessary condition for extreme, Euler's equation, Euler's equation of several variables, invariance of Euler's equation, Motivating problems of calculus of variation, Shortest distance, Minimum surface of revolution, Brachistochrone Problem, Isoperimetric problem, Geodesic.

Unit – III The fixed end point problem for 'n' unknown functions, variational problems in parametric form, Generalization of Euler's equation to (i) 'n' dependent functions (ii) higher order derivatives. Variational problems with subsidiary conditions,

Unit – IV Hamilton's principle, Hamilton's canonical equations, Lagrange's equation from Hamilton's principle Extension of Hamilton's Principle to nonholonomic systems, Application of Hamilton's formulation (Hamiltonian) cyclic coordinates & conservation theorems, Routh's procedure, Hamilton’s equations from variational principle, The principle of least action. Kepler’s law of planetary motion.